Der – Int – Calc – Thm - Gph Worksheet

Key

Derivatives

Basic derivatives: ($f'(x), \frac{dy}{dx}, y'$) **Find** y'.

1) $y = 4$

$y' = 0$

2) $y = 2x^3 + 3x^2 - 5x + 1$

$y' = 6x^2 + 6x - 5$

3) $y = \sin x$

$y' = \cos x$

4) $y = \cos x$

$y' = -\sin x$

5) $y = \tan x$

$y' = \sec^2 x$

6) $y = \csc x$

$y' = -\csc x \cot x$

7) $y = \sec x$

$y' = \sec x \tan x$

8) $y = \cot x$

$y' = -\csc^2 x$

9) $y = \ln x$

$y' = \frac{1}{x}$

10) $y = e^x$

$y' = e^x$

Basic derivatives with chain rule: $y' = operation(u)u'$

1) $y = (x^2 + 3x - 1)^3$

$y' = 3(x^2 + 3x - 1)^2(2x + 3)$

2) $y = \sin 4x$

$y' = 4\cos 4x$

3) $y = \cos x^2$

$y' = -\sin x^2 (2x)$

4) $y = \tan(3x + 1)$

$y' = \sec^2(3x+1)(3)$

5) $y = \cot 2x$

$y' = -\csc^2(2x)(2)$

6) $y = \sec(3x - 4)$

$y' = \sec(3x-4)\tan(3x-4)(3)$

7) $y = \csc(x^2 - 1)$

$y' = -\csc(x^2-1)\cot(x^2-1)(2x)$

8) $y = \ln(2x + 3)$

$y' = \frac{1}{2x+3}(2)$

9) $y = e^{3x^2 - 4}$

$y' = e^{3x^2-4}(6x)$

Product rule using basic derivatives: $y = f \cdot g \quad y' = f'g + g'f$ $y' = f'g + g'f$

1) $y = \ln x \cdot x^2$

$y' = \frac{1}{x}(x^2) + \ln x\,(2x)$
$= x + 2x \ln x$

2) $y = \cos x \cdot e^x$

$y' = -\sin x\, e^x + \cos x\, e^x$

3) $y = \sin x \cdot \tan x$

$y' = \cos x \tan x + \sin x \sec^2 x$

4) $y = \cot x \cdot e^x$

$y' = -\csc^2 x \cdot e^x + \cot x \cdot e^x$

~~or $\frac{1}{\sin x} \cdot \frac{1}{\cos x}$~~ ugly

5) $y = 2x^3 e^x$

$y' = 6x^2 e^x + 2x^3 e^x$

6) $y = \csc x \cdot \sec x$

$y' = -\csc x \cot x \cdot \sec x + \csc x \sec x \tan x$

Quotient rule using basic derivatives: $y = \frac{f}{g}, \quad y' = \frac{gf' - fg'}{g^2}$

1) $y = \frac{\cos x}{e^x}$

$y' = \frac{e^x(-\sin x) - \cos x\, e^x}{(e^x)^2}$

2) $y = \frac{3x^2 - 3x + 1}{\tan x}$

$y' = \frac{\tan x(6x-3) - (3x^2-3x+1)\sec^2 x}{\tan^2 x}$

3) $y = \frac{\ln x}{\sin x}$

$= \frac{\sin x \cdot \frac{1}{x} - \ln x \cos x}{\sin^2 x}$

4) $y = \frac{\csc x}{x^4}$

$y' = \frac{x^4(-\csc x \cot x) - \csc x\,(4x^3)}{x^8}$

5) $y = \frac{e^x}{\cot x}$

$= \frac{\cot x\, e^x - e^x(-\csc^2 x)}{\cot^2 x}$

6) $y = \frac{\sec x}{\ln x}$

$y' = \frac{\ln x(\sec x \tan x) - \sec x \cdot \frac{1}{x}}{(\ln x)^2}$

Special Derivatives

1) $\frac{d}{dx}f(3x^2)$ 2) $\frac{d}{dx}[f(x)+g(x)]$ 3) $\frac{d}{dx}[f(x)\cdot g(x)]$ 4) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$

$f'(3x^2)(6x)$ $f'(x)+g'(x)$ $f'g+g'f$ $\frac{gf'-fg'}{g^2}$

Implicit Differentiation:

3) $3xy^2-5x+12y+2e^y=5$

a) Find $\frac{dy}{dx}$

$3(y^2+x2yy')-5+12y'+2e^y y'=0$

$3y^2+6xyy'-5+12y'+2e^y y'=0$

$y'(6xy+12+2e^y)=5-3y^2$

$y'=\frac{5-3y^2}{6xy+12+2e^y}$

b) Find $\frac{dy}{dt}$

$3\left(\frac{dx}{dt}y^2+x2y\frac{dy}{dt}\right)-5\frac{dx}{dt}+12\frac{dy}{dt}+2e^y\frac{dy}{dt}=0$

$3y^2\frac{dx}{dt}+6xy\frac{dy}{dt}-5\frac{dx}{dt}+12\frac{dy}{dt}+2e^y\frac{dy}{dt}=0$

$\frac{dy}{dt}(6xy+12+2e^y)=5\frac{dx}{dt}-3y^2\frac{dx}{dt}$

$\frac{dy}{dt}(6xy+12+2e^y)=\frac{dx}{dt}(5-3y^2)$

$\frac{dy}{dt}=\frac{5-3y^2}{6xy+12+2e^y}\frac{dx}{dt}$

$\frac{dy}{dt}=\frac{dy}{dx}\cdot\frac{dx}{dt}$

Logarithmic Differentiation:

4) $y=x^{\sin x}$

$\ln y=\ln x^{\sin x}$

$\ln y=\sin x\ln x$

$\frac{1}{y}y'=\sin x\left(\frac{1}{x}\right)+\ln x\cos x$

$y'=y\left(\frac{\sin x}{x}+\ln x\cos x\right)$

$y'=x^{\sin x}\left(\frac{\sin x}{x}+\ln x\cos x\right.$

Fundamental Theorem of Calculus: (derivation of an integral)

5) $\frac{d}{dx}\left(\int_{2x}^{x^2}t^3+\cos t\,dt\right)$

$\left((x^2)^3+\cos x^2\right)(2x)$

$-\left((2x)^3+\cos 2x\right)(2)$

6) $g(x)=2x-7+\int_1^x f(t)\,dt$

$g'=2+f(x)$

Integrals

Basic Integrals (anti derivatives)

$$\int du = u + c \qquad \int k\, f(u)\,du = k\int f(u)\,du$$
$$\int \left(f(u) \pm g(u)\right)du = \int f(u)\,du \pm \int g(u)\,du$$

$\int u^n\ du = \dfrac{u^{n+1}}{n+1} + c$ Power rule good for any power except x^{-1} which is equal to $\dfrac{1}{x}$

examples: $\int x^3\ dx = \dfrac{x^4}{4} + c,\quad \int x^{-3}\ dx = \dfrac{x^{-2}}{-2} + c,\quad \int x^{\frac{1}{2}}\ dx = \dfrac{2x^{\frac{3}{2}}}{3} + c$

1) $\int x^7 dx =$

$\dfrac{x^8}{8} + C$

2) $\int \sqrt[3]{x}\, dx =$

$\int x^{\frac{1}{3}}\, dx$

$\frac{3}{4} x^{\frac{4}{3}} + C$

3) $\int \dfrac{dx}{x^{-3}} =$

$\int x^3\, dx$

$\dfrac{x^4}{4} + C$

U substitution and power rule:

4) $\int \underline{x}\left(2x^2 - 5\right)^3 \underline{dx} =$

$u = 2x^2 - 5$
$du = 6x\,dx$

$\int u^3\ du$

$\frac{1}{6}\ \frac{u^4}{4} + C$

$\frac{1}{24}\left(2x^2 - 5\right)^4 + C$

5) $\int \dfrac{3x^2}{5x^3 - 7} dx =$

$u = 5x^3 - 7$
$du = 15x^2\, dx$
$du = 5(3x^2)\, dx$

$\frac{1}{5}\int \dfrac{du}{u}$

$\frac{1}{5} \ln|u| + C$

$\frac{1}{5} \ln\left|5x^3 - 7\right| + C$

$\int u^{-1}\,du$ *same as* $\int \frac{1}{u}\,du$ *same as* $\int \frac{du}{u} = \ln|u| + c$

6) $\int \frac{1}{t}\,dt =$

$\ln|t| + C$

7) $\int \frac{x\,dx}{x^2-4} =$

$u = x^2 - 4$
$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{u}$

$\frac{1}{2}\ln|u| + C$

$\frac{1}{2}\ln|x^2-4| + C$

8) $\int \frac{1}{2y-3}\,dy =$

$u = 2y - 3$
$du = 2\,dy$

$\frac{1}{2}\int \frac{du}{u}$

$\frac{1}{2}\ln|u| + C$

$\frac{1}{2}\ln|2y-3| + C$

$\int e^u\,du = e^u + c$

9) $\int e^x\,dx =$

$e^x + C$

10) $\int e^{2x}\,dx =$

$u = 2x$
$du = 2\,dx$

$\frac{1}{2}\int e^u\,du$

$\frac{1}{2}e^u + C$

$\frac{1}{2}e^{2x} + C$

11) $\int 3xe^{x^2}\,dx =$

$u = x^2$
$du = 2x\,dx$

$\frac{3}{2}\int e^u\,du$

$\frac{3}{2}e^u + C$

$\frac{3}{2}e^{x^2} + C$

$\int \sin u\,du = -\cos u + c$

12) $\int \sin x\,dx =$

$-\cos x + C$

13) $\int \sin 2x\,dx =$

$u = 2x$
$du = 2\,dx$

$\frac{1}{2}\int \sin u\,du$

$-\frac{1}{2}\cos u + C$

$-\frac{1}{2}\cos 2x + C$

14) $\int x\sin x^2\,dx =$

$u = x^2$
$du = 2x\,dx$

$\frac{1}{2}\int \sin u\,du$

$-\frac{1}{2}\cos u + C$

$-\frac{1}{2}\cos x^2 + C$

$\int \cos u\,du = \sin u + c$

15) $\int \cos x\,dx =$

$\sin x + C$

16) $\int -3\cos 4x\,dx =$

$u = 4x$
$du = 4\,dx$

$\frac{-3}{4}\int \cos u\,du$

$\frac{-3}{4}\sin u + C$

$\frac{-3}{4}\sin 4x + C$

17) $\int x^3\cos 2x^4\,dx =$

$u = 2x^4$
$du = 8x^3\,dx$

$\frac{1}{8}\int \cos u\,du$

$\frac{1}{8}\sin u + C$

$\frac{1}{8}\sin 2x^4 + C$

$\int \sec^2 u\,du$ *same as* $\int (\sec u)^2\,du = \tan u + c$

18) $\int \sec^2 5x\,dx =$

$u = 5x$
$du = 5dx$

$\frac{1}{5}\int (\sec u)^2\,du$

$\frac{1}{5}\tan u + c$

$\frac{1}{5}\tan 5x + c$

19) $\int x^4 (\sec x^5)^2\,dx =$

$u = x^5$
$du = 5x^4\,dx$

$\frac{1}{5}\int (\sec u)^2\,du$

$\frac{1}{5}\tan u + c$

$\frac{1}{5}\tan x^5 + c$

$\int \csc^2 u\,du$ *same as* $\int (\csc u)^2\,du = -\cot u + c$

20) $\int \csc^2(-3x)\,dx =$

$u = -3x$
$du = -3dx$

$-\frac{1}{3}\int \csc^2 u\,du$

$+\frac{1}{3}\cot u + c$

$+\frac{1}{3}\cot(-3x) + c$

21) $\int -2x\csc^2(x^2)\,dx$

$u = x^2$
$du = 2x\,dx$

$-\int \csc^2 u\,du$

$+\cot u + c$

$+\cot x^2 + c$

$\int \sec u \tan u\,du = \sec u + c$

22) $\int \sec 5x\,\tan 5x\,dx$

$u = 5x$
$du = 5dx$

$\frac{1}{5}\int \sec u \tan u\,du$

$\frac{1}{5}\sec u + c$

$\frac{1}{5}\sec 5x + c$

23) $\int x^5 \sec x^6 \tan x^6\,dx$

$u = x^6$
$du = 6x^5\,dx$

$\frac{1}{6}\int \sec u \tan u\,du$

$\frac{1}{6}\sec u + c$

$\frac{1}{6}\sec x^6 + c$

$\int \csc u \cot u\,du = -\csc u + c$

24) $\int \csc 3x \cot 3x\,dx$

$u = 3x$
$du = 3dx$

$\frac{1}{3}\int \csc u \cot u\,du$

$-\frac{1}{3}\csc u + c$

$-\frac{1}{3}\csc 3x + c$

Calculator

1) Find the zero's: (quadratic formula)

$$f(x) = 2x^2 - 5x - 8$$

$X = -1.108$
$X = 3.608$

2) Find the zero(s) (no quadratic, calculate zero in window)

$$f(x) = 2x^3 - 3x^2 + 2x - 5$$

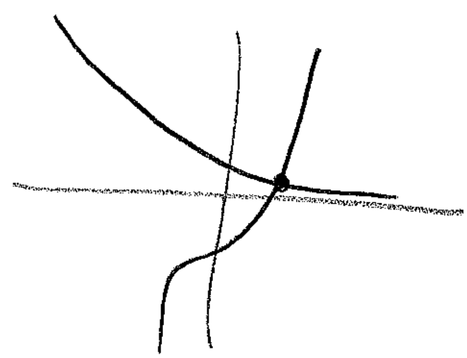
$X = 1.747$ OR 1.746

3) Find the intersection:

Can be written 3 different ways: In 3 you only get the x coordinate (plug in the function to get the y coordinate)

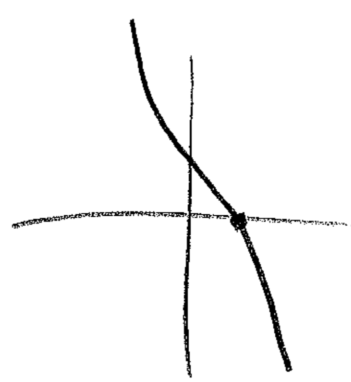
(1.288, .276)
.275

a) $y = 2x^3 - 4$
$y = e^{-x}$



CALC Intersection

b) $2x^3 - 4 = e^{-x}$



c) $0 = e^{-x} - 2x^3 + 4$

Calc zero $x = (1.288)$

$y = 1E^{-13} = 0$

4) Limits:

$$\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}$$

0^- 0^+

x	-.1	-.01	-.001		.001	.01	.1
value	.25158	.25016	.25002	$\boxed{.25}$	.24998	.24984	.24846

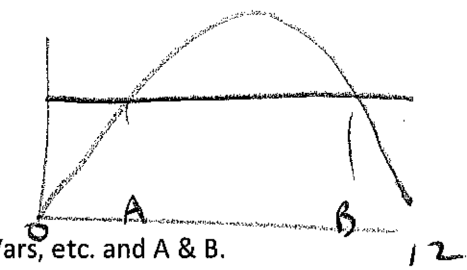
$$\lim_{x\to -\infty}\left(\frac{x}{2}+\sqrt{\frac{x^2}{4}+x}\right)$$

x	-10	-100	-1000	
value	-1.127	-1.01	-1.001	$\boxed{-1}$

5) On the interval $[0,12]$, find the area of the region bound by the curves

$y=12\sin\left(\frac{x}{4}\right)$ & $y=7$.

Radian Mode
Window 0, 12
Zoom Fit



Find points of intersection, store the x values in A & B. Use Math 9, Vars, Y-Vars, etc. and A & B.

$$A=\int_0^A 7-12\sin\frac{x}{4}\,dx+\int_A^B 12\sin\frac{x}{4}-7\,dx+\int_B^{12} 7-12\sin\frac{x}{4}\,dx = 38.256$$

$A = 2.4913063$
$B = 10.6675064$

6) Given $v(t)=12\sin\left(\frac{x}{4}\right)$, $0\le t\le 20$, $x(0)=-14$

a) Find $a(7)$. Use in window use Calc 6 and type in 7.

$v'(7)=a(7)=-.535$

b) Find position $x(15)$

Initial condition formula

$$x(15) = -14+\int_0^{15} 12\sin\frac{x}{4}\,dx = 73.387$$

c) Find the distance traveled on the interval $[2,16]$

$$\text{Dist} = \int_2^{16} |v(t)|\,dt = 106.749$$

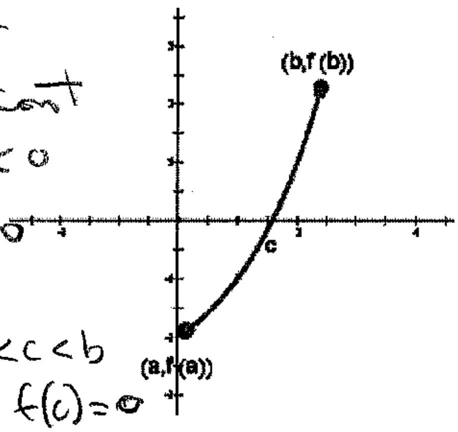
d) Find the displacement on the interval $[4,14]$

$$\text{Disp} = \int_4^{14} v(t)\,dt = 70.884$$

Theorems – Definitions – Graphs

Write a theorem or definition that each graph is trying to display.

1)



IVT

f is cont

$f(a) < 0$

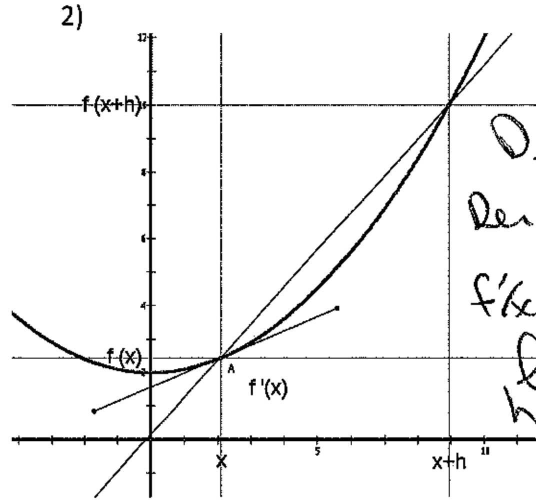
$f(b) > 0$

Some c

s.t. $a < c < b$

where $f(c) = 0$

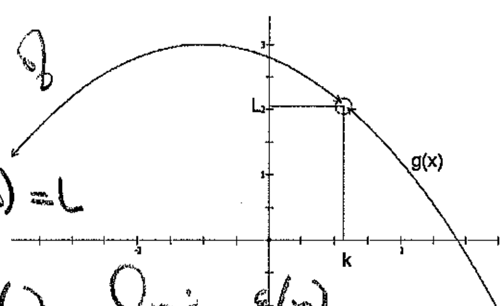
2)



Definition of a Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3)



Definition of a limit

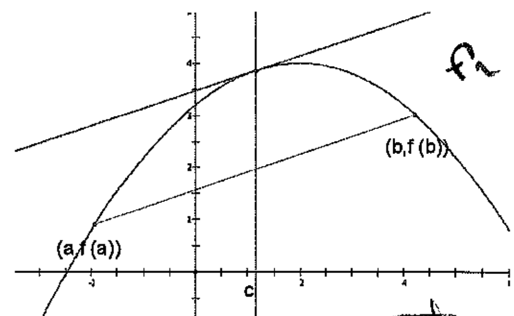
$$\lim_{x \to k} g(x) = L$$

$$\Rightarrow \lim_{x \to k^-} g(x) = \lim_{x \to k^+} g(x)$$

$$L = L$$

4)

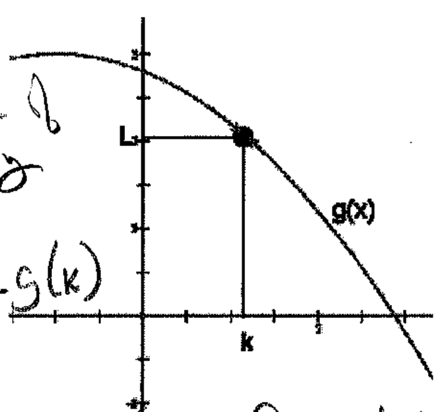
MVT for Derivatives



$$m_{tangent} = m_{secant}$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

5)



Definition of Continuity

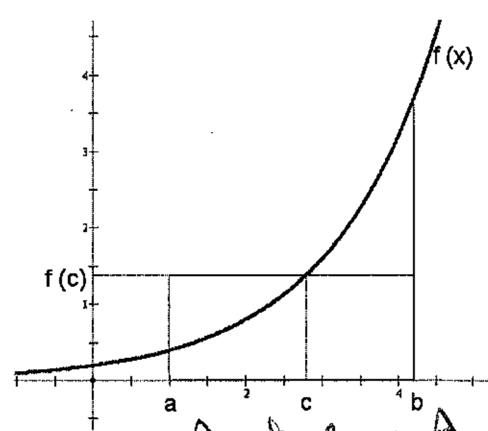
$$\lim_{x \to k} g(x) = g(k)$$

&

$$\lim_{x \to k^-} g(x) = g(k) = \lim_{x \to k^+} g(x)$$

6)

MVT for Integrals

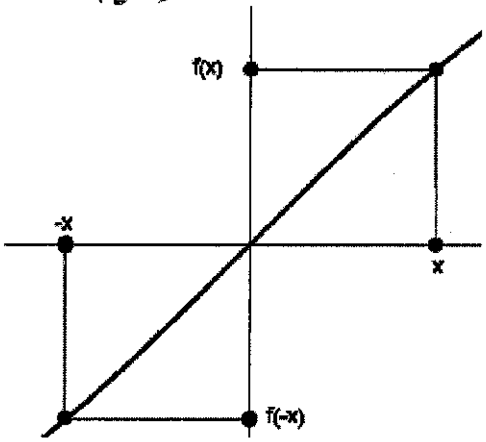


$A_{rectangle} = A$ under curve

$$(b-a) f(c) = \int_a^b f(x)\,dx$$

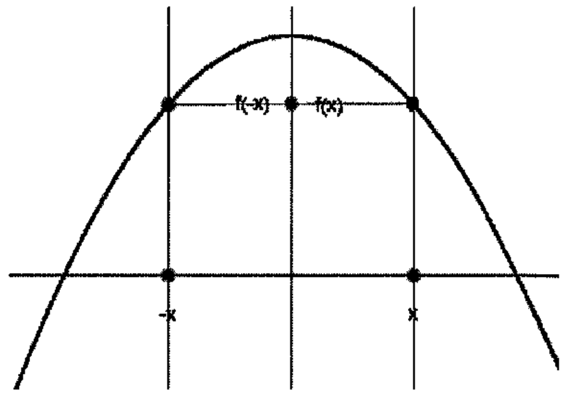
7) ODD function

$f(x) = -f(-x)$



8) Even function

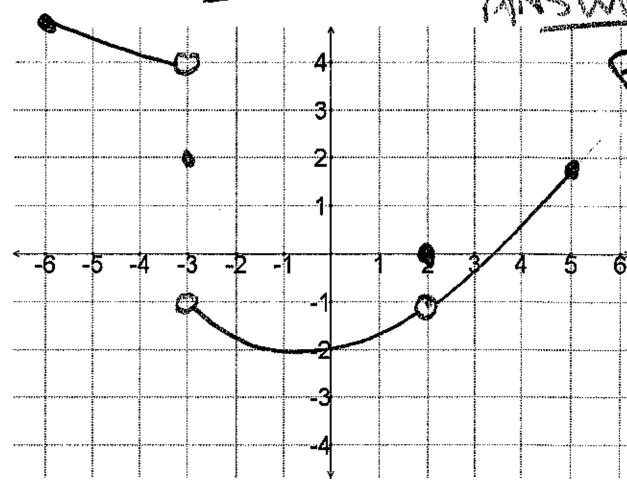
$f(x) = f(-x)$



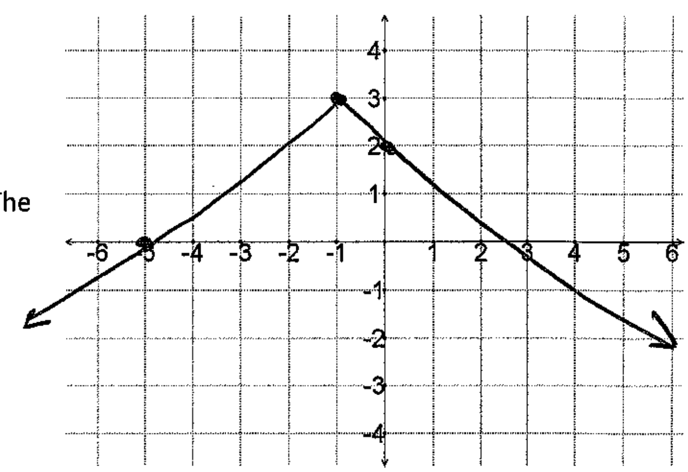
MORE THAN 1 CORRECT ANSWER FOR 9-16

Sketch the function that satisfies the given information.

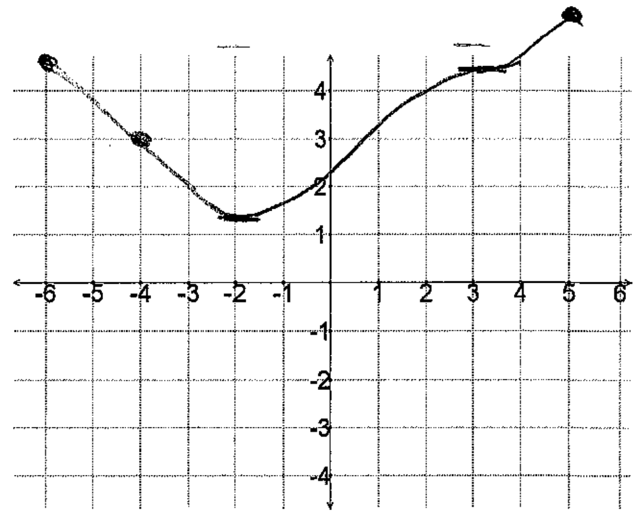
9) Domain $[-6,5]$ $\lim_{x\to -3^+} f(x) = -1$, $\lim_{x\to -3^-} f(x) = 4$, $f(-3) = 2$, $\lim_{x\to 2} f(x) = -1$, $f(2) = 0$



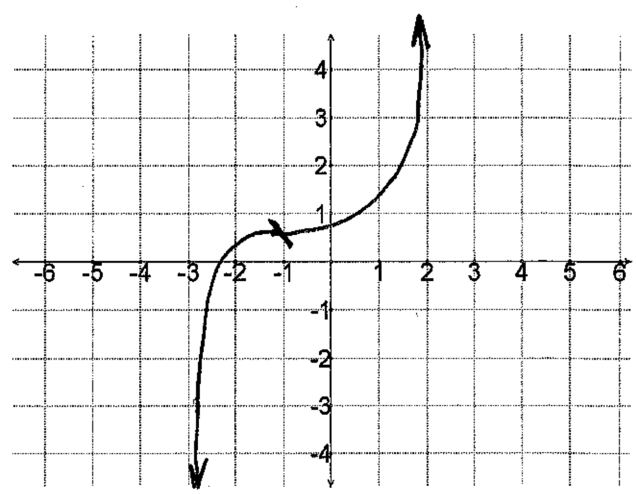
10) Domain $(-\infty,\infty)$, $f(x)$ is continuous at $x = -1$, but not differentiable and $f(-1) = 3$. The x-intercept is -5 and the y-intercept is 2



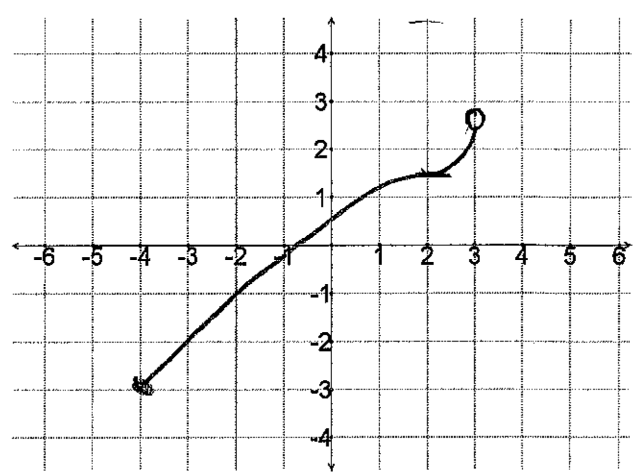
11) $f(-4)=3$, $f'(-2)=0$, $f'(3)=0$, $f'(x)<0$ in the interval $(-6,-2)$, $f'(x)>0$ in the intervals $(-2,3)$ and $(3,5)$, f is continuous on the interval $[-6,5]$



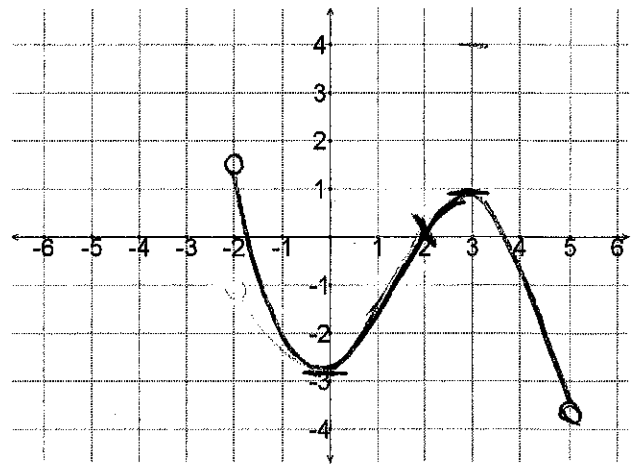
12) f is continuous on the interval $(-3,2)$, $\lim_{x\to-3^+} f(x)\to-\infty$, $\lim_{x\to2^-} f(x)\to\infty$, $f''(x)<0$ in the interval $(-3,-1)$ and $f''(x)>0$ in the interval $(-1,2)$.



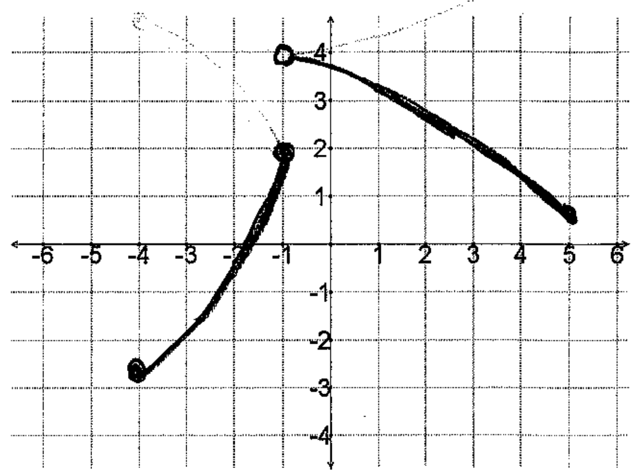
13) The domain of f is $[-4,3)$, $f'(x)>0$ for all x, $f'(2)=0$, there is an absolute minimum of -3, but no absolute maximum.



14) The domain of f is $(-2,5)$, there is a relative (local) minimum at $x=0$ and a relative maximum at $x=3$ and a point of inflection at $x=2$, but no absolute maximum or absolute minimum



15) The domain of f is $[-4,5]$, $\lim_{x\to -1^-} f(x)=2$, $\lim_{x\to -1^+} f(x)=4$, $f(-1)=2$, on the interval $(-4,-1)$ both $f'(x)\ and\ f''(x)>0$, on the interval $(-1,5)$ both $f'(x)\ and\ f''(x)<0$.



16) The domain of f is $[-4,5]$, there are 3 critical points at $x=-2,1,3$, absolute (global) maximum at $x=-4$, absolute (global) minimum at $x=1$, relative maximum at $x=3$, but no relative extreme at $x=-2$.